Lecture 14: Hypothesis Testing

For each question, write the R code you would use and record the result when appropriate. Complete this in an RMarkdown document so that you will have the code/explanations for your personal records. Answer in complete sentences when asked to explain. Unless otherwise stated, use a significance level of $\alpha = 0.05$ and use a two-sided hypothesis test.

Part 1: The Big Idea of Hypothesis Testing

1. In your own words, explain how hypothesis testing is related to confidence intervals.

2. A school claims that the average score on a placement exam is 82.3 points. A researcher wants to test whether the true mean score is different from 82.3 points.

   (a) What is the population parameter in this problem?

   (b) State the null hypothesis.

   (c) State the alternative hypothesis.

   (d) Is this a one-sided or two-sided test? Explain briefly.

Part 2: Possible Conclusions

3. In hypothesis testing, there are only two possible conclusions.

   (a) What does it mean to reject the null hypothesis?

   (b) What does it mean to fail to reject the null hypothesis?

   (c) Why should we avoid saying that we "accept" the null hypothesis?

Part 3: Significance Levels and Rejection Regions

4. Suppose we are using a two-sided test with $\alpha = 0.05$ and the standard normal distribution.

   (a) Use `qnorm()` to find the lower critical value.

   (b) Use `qnorm()` to find the upper critical value.

   (c) What values of the test statistic would cause us to reject the null hypothesis?

   (d) What values of the test statistic would cause us to fail to reject the null hypothesis?

5. For each test statistic below, determine whether it falls in the rejection region for a two-sided test with $\alpha = 0.05$.

   (a) $z = 2.21$

   (b) $z = -1.35$

   (c) $z = -2.08$

   (d) $z = 0.74$

Part 4: Hypothesis Test when σ is Known

6. The nutrition label on a bag of potato chips says that a one-ounce serving has 130 calories. Assume the population standard deviation is known to be 17 calories. A random sample of 35 bags has a sample mean of 134 calories. We want to know if there is evidence that the nutrition label does not provide an accurate measure of the calories.

   (a) State the null hypothesis.

   (b) State the alternative hypothesis.

   (c) Identify the sample mean, hypothesized mean, population standard deviation, and sample size.

(d) Compute the standard error.

(e) Compute the test statistic.

(f) Use `qnorm()` to find the rejection region for a two-sided test with $\alpha = 0.05$.

(g) Based on the rejection region, should we reject or fail to reject the null hypothesis?

(h) Use `pnorm()` to calculate the two-sided p-value.

(i) Compare the p-value to $\alpha = 0.05$.

(j) Write a conclusion in context.

Part 5: Hypothesis Test when σ is Unknown

7. We will now look at the `course_grade` variable in the `exam_grades` dataset within the `openintro` library. Suppose a department claims that the average course grade is 74 points. We want to test whether the true mean course grade is different from 74 points.

   (a) Load the `openintro` library and display the first 6 observations of `exam_grades`.

   (b) What does one row of the dataset appear to represent?

   (c) Is `course_grade` quantitative or categorical? Explain briefly.

8. **Step 1: Gathering the Information**

   (a) Find the sample mean of `grades`.

   (b) Find the sample standard deviation of `grades`.

   (c) Find the sample size of `grades`.

   (d) State the degrees of freedom for this test.

   (e) Explain why we should use the t-distribution instead of the standard normal distribution for this problem.

9. **Step 2: Setting Up the Hypothesis Test**

   (a) State the null and alternative hypothesis.

   (b) Is this a one-sided or two-sided test? Explain briefly.

10. **Step 3: Calculating the Test Statistic and p-value Manually in R**

   (a) Compute the test statistic.

   (b) Use `qt()` to find the rejection region for a two-sided test with $\alpha = 0.05$.

   (c) Based on the rejection region, should we reject or fail to reject the null hypothesis?

   (d) Use `pt()` to calculate the two-sided p-value and compare the p-value to $\alpha = 0.05$.

   (e) Write a conclusion in context.

11. **Step 4: Checking with `t.test()`**

    (a) Use `t.test()` to run the same hypothesis test.

    (b) Do the test statistic and p-value match what you found manually?

    (c) Why is it useful to know how to do the test manually if R can do it with `t.test()`?

Part 6: Connecting Confidence Intervals and Hypothesis Tests

12. In the previous part, you used `t.test()` to test whether the true mean course grade was different from 74 points. We are going to build a confidence interval and see how they are related to hypothesis testing.

    (a) Construct a 95% confidence interval for the true mean course grade.

    (b) Is the claimed value of 74 inside or outside of the 95% confidence interval?

    (c) Based only on the confidence interval, does 74 seem like a reasonable value for the true mean course grade? Explain briefly.

(d) Look back at your hypothesis test conclusion from Part 5. Did you reject or fail to reject the null hypothesis?

(e) How does your hypothesis test conclusion match what you saw from the confidence interval?

13. Now suppose someone claimed that the true mean course grade was 72 points instead of 74 points.

(a) Is 72 inside or outside of your 95% confidence interval?

(b) Based only on the confidence interval, would 72 seem like a reasonable value for the true mean course grade? Explain briefly.

(c) Without running another hypothesis test, would you expect to reject or fail to reject the null hypothesis if $H_0 : \mu = 72$? Explain how you know.

14. In one or two sentences, explain the connection between a 95% confidence interval and a two-sided hypothesis test with $\alpha = 0.05$.